

A Simple Explanation for DAMA with Moderate Channeling

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We consider the possibility that the DAMA signal arises from channeled events in simple models where the dark matter interaction with nuclei is suppressed at small momenta. As with the standard WIMP, these models have two parameters (the dark matter mass and the size of the cross-section), without the need to introduce an additional energy threshold type of parameter. We find that they can be consistent with channeling fractions as low as about $\sim 15\%$, so long as at least $\sim 70\%$ of the nuclear recoil energy for channeled events is deposited electronically. Given that there are reasons not to expect very large channeling fractions, these scenarios make the channeling explanation of DAMA much more compelling.

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I. INTRODUCTION

The annual modulation signal seen by the Dark Matter (DAMA) collaboration, now significant at the 8σ level [1, 2], provides an intriguing puzzle. If one assumes that this signature is due to a standard weakly interacting massive particle (WIMP) elastically scattering off nuclei, then it follows that other direct detection experiments should also have seen large numbers of signal events. This is, however, not the case. As a result, one is left to explore dark matter scenarios going beyond the standard elastic WIMP paradigm. Indeed, it is not currently possible to rule out a dark matter explanation for DAMA in a model independent way. The list of known possibilities still consistent with all available data, however, is in fact rather limited.

One scenario which has received a great deal of attention is that of inelastic dark matter [3]. In this picture, one assumes that in each scattering event the dark matter particle makes a transition to a higher energy state, separated from the original by a splitting δ . This splitting effectively removes scattering events at sufficiently small nuclear recoil energies, where standard WIMP event rates become large and cause the greatest tension with data. Another possibility is that the events at small recoil energies become suppressed by a dynamical form factor [4, 5]. Yet another option is that the scattering proceeds through a resonance [6].

In all of the above cases, however, great care must be taken to ensure that the DAMA signal is present, without resulting in an over-abundance of events at other experiments. In inelastic dark matter, a splitting is added solely to remove events at experiments with light nuclei. This splitting cannot be too small, lest the low energy suppression be insufficient, but it also cannot be too large, or else the DAMA signal disappears. Hence, for any given dark matter mass, δ must fall in a narrow range, and additional model building is required to motivate the coincidence associated with its size.

In the dynamical form factor scenario, careful model building must be undertaken in order to ensure that the

required low energy suppression is sufficiently rapid. In explicit models constructed so far, here too there is a parameter (analogous to δ) that must be carefully chosen to make the DAMA signal consistent with the null experiments.

While such model building avenues are currently still viable for explaining DAMA, perhaps a nicer resolution would be obtained if there were a reason found for DAMA simply being the most sensitive direct detection experiment at low recoil energies, where WIMP signals tend to be peaked. In fact, “channeling” has already been proposed as an explanation of this type [7, 8, 9, 10, 11]. Most direct detection experiments, including DAMA, do not measure the complete recoil energy of a nucleus in a scattering event, but only some fraction of it. In particular, DAMA measures only the fraction deposited electromagnetically. The complete recoil energy is then inferred through multiplication by an experimentally determined “quenching factor”. DAMA, modeling their quenching factor to be the same for all recoiling Iodine nuclei, determines it to have a value of 0.09. The idea behind channeling is that, due to the crystal structure of the detector, some fraction of low energy events may actually have a quenching factor closer to one. This is the expectation for nuclear recoils which travel sufficiently along an axis of the crystal. For such “channeled” events, a nuclear recoil which was thought to have had an energy of, say ~ 20 keV, actually had an energy closer to ~ 2 keV.

In this letter, we will reconsider the possibility of explaining the DAMA data through the assumption that some fraction of recoiling nuclei are being channeled. Since we believe that current theoretical estimates for the amount of channeling are most likely not reliable, our goal will be to try to explain the DAMA data using as little channeling as possible. We will show that unlike the case of a standard WIMP, the simplest momentum dependent couplings can allow for an appreciable decrease in the amount of channeling required to explain the data, to as little as $\sim 10\%$. In addition, for channeling fractions of at least $\sim 15\%$, these couplings can allow for channeled events to have a quenching factor as small as

$\sim 70\%$ and still satisfy all constraints.

No complicated model building is needed to arrange for the required momentum dependence. For example, it is sufficient to have a neutral dark matter particle, X , whose leading interaction with a new GeV mass gauge boson A_μ therefore becomes the lowest dimension gauge-invariant interaction $\mathcal{L} = \frac{i}{\Lambda^2} F_{\mu\nu} \partial^\mu X^* \partial^\nu X$. Kinetic mixing between A_μ and hypercharge then couples X to nuclei in a momentum dependent fashion. The mass of the gauge boson and the scale $\Lambda \sim \text{GeV}$ are both absorbed into the overall scattering cross-section, and so constitute a single parameter.

The scenario discussed here therefore offers a solution to the DAMA puzzle that does not require extra energy threshold parameters, and is viable with more conservative and realistic versions of channeling. Finally, in the event that heavy element experiments such as CRESST [12, 13], XENON [14], and KIMS [15] fail to find evidence for dark matter in the near future, this scenario may remain as an attractive viable option.

II. REVIEW

The event rate per unit detector mass per unit recoil energy at a dark matter direct detection experiment is given by (see e.g. [16, 17])

$$\frac{dR}{dE_R} = N_T \frac{\rho_{\text{DM}}}{m_{\text{DM}}} \int_{v_{\min}} d^3v f(\vec{v}) v \frac{d\sigma}{dE_R}. \quad (1)$$

In this expression, N_T is the number of target nuclei per unit mass, ρ_χ is the local dark matter density and m_{DM} is the mass of the dark matter particle. The integration is over the distribution of dark matter velocities $f(\vec{v})$ relative to the Earth; the lower bound v_{\min} is given by the minimum velocity a dark matter particle must have in order to cause a nuclear recoil with energy E_R . In particular, we have $v_{\min} = \frac{1}{\mu} \sqrt{\frac{m_N E_R}{2}}$, where m_N is the mass of the recoiling nucleus, and μ is the dark matter-nucleus reduced mass.

The cross section for a dark matter particle to scatter off of a nucleus with charge Z and atomic number A is given by

$$\frac{d\sigma}{dE_R} = \frac{m_N}{2v^2} \frac{\sigma_p}{\mu_n^2} \frac{(f_p Z + f_n(A - Z))^2}{f_p^2} F_N^2(E_R) F_{\text{DM}}^2(E_R). \quad (2)$$

Here μ_n is the dark matter-nucleon reduced mass, f_n and f_p are the relative coupling strengths to protons and neutrons, and F_N is the nuclear form factor. F_{DM} is a possible form factor coming from the dark matter sector, which is equal to 1 for a standard WIMP, and in general is normalized to be 1 at momentum transfer $q = \sqrt{2m_N E_R} = 30 \text{ MeV}$. σ_p is an overall constant that would be the dark matter-proton cross section in the case where $F_{\text{DM}} \equiv 1$. For simplicity we will take $f_n = 0$ in

this letter, corresponding, for example, to scattering taking place through a new $U(1)$ gauge symmetry which mixes with hypercharge.

The form of the dark matter velocity distribution is significantly uncertain. It is generally considered that a Boltzmann distribution $\propto e^{-v^2/\bar{v}^2}$ in the galactic rest frame is a reasonable starting point. We will consider the cases $\bar{v} = 170 \text{ km/s}$, 220 km/s , and 270 km/s , and take the distributions to be cut off at an escape velocity v_{esc} whose value we shall take to be 500 km/s [18].

We will parameterize the velocity v_e of the Earth's galactic motion, as well as the nuclear form factors, as in [4].

III. CHANNELING

Channeling of fast-moving ions through crystal lattices is a well-established physical phenomenon with an extensive literature. For a relatively recent review, see [19]. The basic theory, developed by Lindhard in the sixties [20], details under what conditions an ion will become guided moving along a particular axis or within a particular plane by the cumulative effects of soft collisions with the lattice atoms. In this theory, a channeled ion never gets close enough to an individual atom to lose a significant amount of energy from elastic scattering. Typically, such an ion will instead lose most of its energy by electronically exciting the atoms it passes.

If a WIMP strikes a nucleus in the DAMA detector, it is ejected from its position in the NaI lattice (dragging along most of its electrons [35]) and becomes a fast-moving ion. As pointed out by Drobyshevski [8], this recoiling ion could in principle become channeled, making DAMA sensitive to much lower energies than originally expected.

There are two crucial questions here: 1) What is the probability for a recoiling ion to become channeled at $O(1 \sim 10) \text{ keV}$ energies in NaI?, and 2) On average, what fraction of the recoil energy of such channeled ions is lost electronically? In [21], the DAMA collaboration made a first attempt at answering these questions, and using Lindhard's theory, estimated the effects to be significant. For Na and I ions at 3 keV , they found channeling probabilities of roughly 30%, and took quenching factors of almost 100%.

However, rigorous experimental and theoretical studies of this regime in NaI are lacking, and the full physical situation remains far from clear. At DAMA energies, calculation of the critical trajectories for channeling falls outside the realm of applicability of the Lindhard theory, as the details of the collective lattice potential become important. Other novel complications appear, as well. For example, a recoiling nucleus necessarily originates at a lattice site, and thus if it is aimed towards a crystal axis or plane, its trajectory will tend to lie in the direction of its neighbors. It might then immediately undergo scattering away from the intended channel [36]. Even

if a recoiling ion manages to find itself channeled after a few atomic scatterings with modest energy loss, it is not obvious that the subsequent electronic energy losses will dominate for such a low-energy channeled ion (see, e.g. [22]). Above all else, it is clear that an experimental study of the channeling of recoiling nuclei in NaI would be extremely valuable.

In the rest of this paper, we will take an agnostic viewpoint towards these issues, and simply parametrize the possibility of a population of recoiling ions depositing an anomalously large fraction of its energy electronically. In the end, even a somewhat large non-gaussian tail of the event-by-event quenching factor can lead to the effects predicted by Drobysheski, and channeling need not even be the primary cause of this. Regardless of its origin, we will continue to call this hypothetical effect “channeling” for continuity.

Should an effect like this occur at the level claimed by DAMA, the implications for their observed annual modulation at ~ 3 keV would be quite dramatic. For light WIMPs with standard SI interactions, the absolute and modulating spectra peak sharply at low energies, and DAMA becomes capable of directly sampling part of this otherwise unobservable region at $E_R \sim 3$ keV. This causes a portion of the DAMA allowed region in the (m, σ_N) plane to edge past constraints from CDMS [23, 24, 25] and XENON [7, 9]. On the other hand, as we will see in the next section, even a modest relaxation of either the assumed channeling fraction or its effective quenching factor will move all of the allowed region back into conflict with these experiments.

A simple, more robust alternative is a dark matter particle with momentum-dependent interactions due to form factors or other explicit E_R -dependence in the scattering matrix elements. As we will see below, even modest amounts of channeling could make simple form factors in the dark sector a viable explanation for all of the present data.

IV. REQUIRED CHANNELING FRACTION AND SHAPE

If the DAMA signal is due to channeled events, then the constraints from many other direct detection experiments can be avoided because most of their energy range would correspond to dark matter events with velocity above the local escape velocity. It is useful to convert the energy range of experiments into a range in momentum transfers q , with q_{\min} the lowest momentum transfer probed. Then a given experimental constraint is completely evaded for dark matter masses below some critical value, $m_{\text{DM}}^{-1} = 2(v_{\text{esc}} + v_e)/q_{\min} - m_N^{-1}$. In particular, it is possible to choose m_{DM} so that all of the CDMS-Ge and CRESST-W range in q is above the escape velocity, while all of the DAMA signal region is below it. The most significant constraints then become CDMS-Si and, depending on its quenching factor, XENON. In addition, it

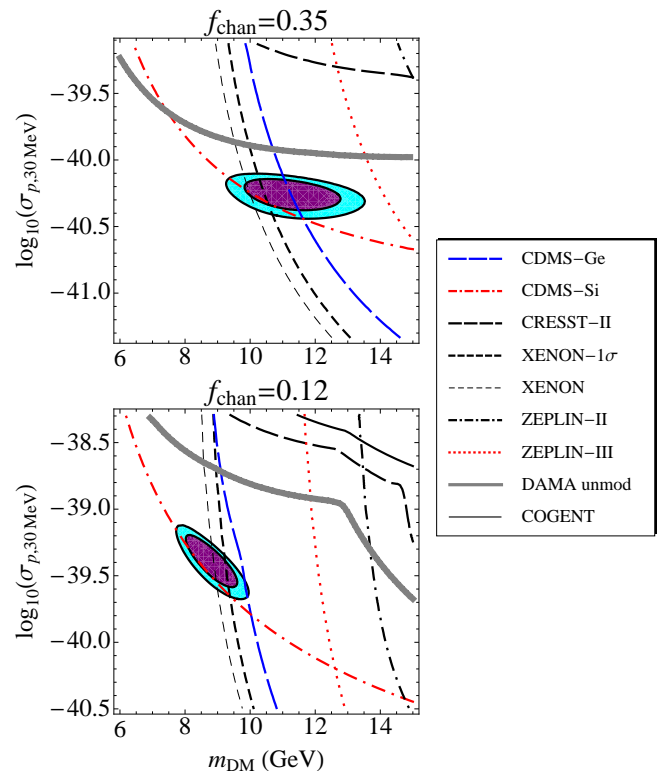


FIG. 1: Constraint plots for channeled events, with an energy-independent channeling fraction. The bottom plot shows constraints on models with a momentum-dependent $F_{\text{DM}}(q) \propto q^2$ coupling, and the top assumes a standard (i.e. momentum-independent) coupling. DAMA 90% and 99% contours are shown. The f_{chan} 's above are chosen to be the minimum values consistent with all constraints. Obviously, larger values of f_{chan} leave more parameter space open.

is important that values of m_{DM} below the critical value still give a decent fit to the DAMA spectrum. We will see that model-dependent assumptions can affect the quality of the fit significantly, allowing or disfavoring smaller dark matter masses.

Because the critical mass for a given experiment depends on q_{\min} , and event rates tend to rise sharply at decreasing q , constraints are very sensitive to even modest uncertainties in quenching factors, which relate the experimental light yield to the actual recoil energy. XENON in particular has measured its quenching factor to be $15.5\% \pm 1.2\%$ (weighted average over [26, 27, 28], but see also [29], which indicates that the experiments may be underestimating their uncertainties). We will therefore present constraints from XENON using both $q_{\text{XENON}} = 15.5\%$ and the lower 1σ value 14.3% (“XENON- 1σ ” for short).

There are a number of directions in which the parameter space opens up if one takes into account uncertainties or variations in models. The quality of the fit

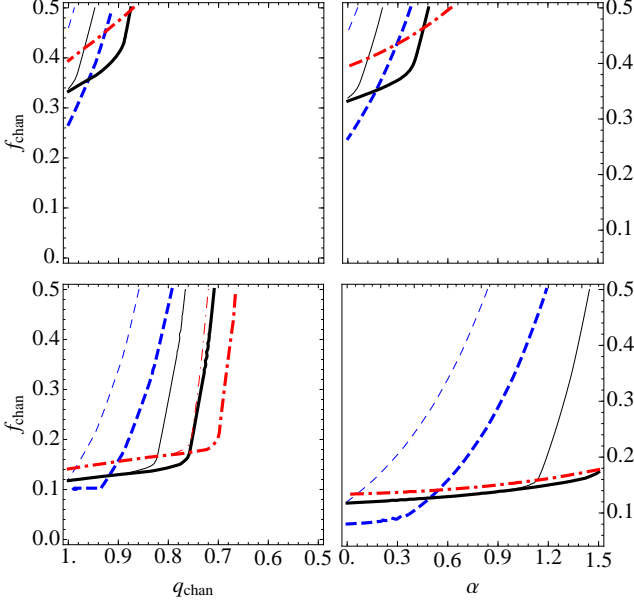


FIG. 2: Minimum channeling fractions allowed at $E_R = 3$ keV depending on the channeled quenching factor q_{chan} and the channeling fraction energy-dependence, $f(E_R) \propto E_R^{-\alpha}$. Constraints are shown for $v_0 = 170, 220, 270$ km/s in blue dashed, black solid, red dot-dashed, respectively, and $q_{\text{XENON}} = 15.5\%$ (14.3%) in thin (thick) lines. The top panels are for a momentum-independent coupling and the bottom are for $F_{\text{DM}} \propto q^2$. Left (right) has $\alpha = 0$ ($q_{\text{chan}} = 1$). The $v_0 = 270$ km/s constraint in the lower right plot is identical for the two values of q_{XENON} we consider.

to DAMA depends on the shape of the predicted spectrum, and including an additional dark matter form factor $F_{\text{DM}}(q) \sim q^2$ allows a range of smaller dark matter masses.

It is important to take into account that the masses favored by DAMA are also sensitive to the energy-dependence of the channeling fraction. Most if not all studies of channeled events assume a channeling fraction of the form $f_1(E_R) = \frac{e^{-E_R/40}}{1+0.65E_R}$ or something very close [9, 10, 30], which over the range $2\text{ keV} < E_R < 6\text{ keV}$ relevant at DAMA is approximately $\propto E_R^{-0.75}$. However, as we have discussed, this estimate is based on a relatively simple analytic approximation due to Lindhard, and for example if the channeling fraction is saturated at the relevant low recoil energies, then the profile could instead be approximately constant [37]. Such a profile would favor somewhat smaller masses, which in turn push more of the null experiments' energy ranges above the escape velocity. Figure 1 shows the 90% constraints on a WIMP and the 90%, 99% DAMA fit contours if the channeling fraction is energy-independent. A channeling fraction of $\approx 35\%$ is the minimum value required to have some region of parameter space allowed by all constraints. The situation is even better with an additional dark matter form factor $F_{\text{DM}} \propto q^2$. In this case, much of the DAMA fa-

n	flat profile	Lindhard profile
$v_0 = 170$ km/s		
0	45.7%, 11.8 GeV, (7.41)	> 100%, -, -
1	26.5%, 10.6 GeV, (6.35)	77.3%, 11.6 GeV, (6.72)
2	12.3%, 9.58 GeV, (5.57)	40.7%, 10.3 GeV, (5.86)
4	3.71%, 8.75 GeV, (5.32)	4.21%, 8.77 GeV, (5.02)
$v_0 = 220$ km/s		
0	35.1%, 9.97 GeV, (7.87)	> 100%, -, -
1	20.4%, 9.16 GeV, (6.56)	42.0%, 9.59 GeV, (7.09)
2	12.5%, 8.37 GeV, (6.30)	14.3%, 8.92 GeV, (6.07)
4	4.96%, 7.64 GeV, (9.14)	5.43%, 7.76 GeV, (7.27)
$v_0 = 270$ km/s		
0	40.9%, 8.96 GeV, (7.49)	57.6%, 9.64 GeV, (8.03)
1	23.3%, 8.15 GeV, (6.99)	27.9%, 8.75 GeV, (6.79)
2	13.9%, 7.76 GeV, (7.55)	15.4%, 7.96 GeV, (7.19)
4	4.74%, 6.98 GeV, (16.2)	5.90%, 7.48 GeV, (11.5)

TABLE I: Minimum channeling fractions at $E_R = 3$ keV allowed at 90% by all constraints for various values of v_0 and form factors $F_{\text{DM}}(q) \propto q^n$. Also shown are the dark matter mass at the minimum channeling fraction and, in parentheses, the χ^2_{DAMA} for the best fit to the DAMA spectrum. In all cases, $q_{\text{XENON}} = 15.5\%$.

vored masses are below the critical value m_{DM} where no events are predicted at XENON or CDMS-Ge, and the minimum channeling fraction allowed becomes $\approx 12\%$. In all of the allowed parameter space, iodine scattering always dominates over sodium scattering at DAMA [38].

Our analysis assumptions are as in [4] except for XENON, where we vary the quenching factor, CRESST-II, where we use only the 2007 run, and CDMS-Si, where we have taken an effective exposure of 12 kg days (from March 25 - Aug 8) and no events. Also, one of the ten potential XENON events was identified by that collaboration as resulting from instrumental error, and we do not include it in calculating our constraints.

Finally, as usual, uncertainty in the halo model has a significant impact on the model constraints. We emphasize that it is difficult to know precisely how to interpret model constraints without better knowledge of the allowed range of dark matter halo velocity distributions. The most significant effect of different halo models tends to be whether the distribution is tighter or broader, so one can qualitatively consider the effect of different halo models by changing the average rotational velocity parameter v_0 . In Table I, we consider the effect of a modest change in the halo distribution by considering Maxwellian distributions with $v_0 = 170$ and 270 km/s in addition to the standard $v_0 = 220$ km/s. We present there the minimum channeling fraction at $E_R = 3\text{ keV}$ allowed by demanding consistency at 90% with all experiments. Note that, while a higher power of q in the form factor decreases the required channeling fraction, one may not continue indefinitely to higher powers since the best fit to DAMA eventually worsens. Furthermore,

when the strongest constraint is XENON, larger values of v_0 are favored, since the broader velocity distribution pushes the DAMA favored region to lower masses. However, when CDMS-Si is the strongest constraint, lower values of v_0 are favored, since larger dark matter masses enhance the ratio of the reduced mass μ at iodine vs. at silicon.

Figure 2 presents the effect from different possibilities for the details of channeling itself, for three halo models ($v_0 = 170, 220, 270$ km/s) and XENON quenching factors ($q_{\text{XENON}} = 15.5\%, 14.3\%$). Since it is unlikely that 100% of the energy of channeled events gets deposited electronically, we consider lower values for this “channeled quenching factor” q_{chan} [39]. For a standard WIMP coupling, relatively high values $q_{\text{chan}} \gtrsim 90\%$ or large channeling fractions $\gtrsim 30\%$ are necessary. However, for a q^2 form factor, even $q_{\text{chan}} \approx 70\%$ may be sufficient with as little as 15% of events channeled. We also show the effect of varying the energy profile of the channeling fraction, parameterized as $f_1(E_R) \propto E_R^{-\alpha}$. For a standard WIMP

coupling, the minimum channeling fraction required at $E_R = 3$ keV depends significantly on α , and the profile must be fairly flat to allow $f_{\text{chan}} < 50\%$. Again, the situation is much improved with a form factor. For $F_{\text{DM}} \propto q^2$, the dependence on α is much weaker, and relatively large α ’s still allow $f_{\text{chan}} \sim 10\%$.

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 - [35] In an intuitive semiclassical picture due to Migdal [34], electrons orbiting at velocities greater than the nucleus’s recoil velocity will be largely unaffected by the scattering. For the recoil velocities relevant at DAMA, this picture applies for almost all of the electrons in both Na and I atoms, and thus most of the electrons will remain in their original orbitals.
 - [36] Within the context of the Lindhard theory, an ion will only end up channeled if it does not penetrate too close to a guiding string or plane of lattice atoms. This criterion is manifestly violated if the starting position of the ion is already within this string or plane.

- [37] An amusing possibility is that the turn-over in the DAMA spectrum at low energies could even be due to a turn-over in the channeling fraction, if channeling became less effective below $E_R \sim 3\text{keV}$.
- [38] We have checked that even if COGENT and KIMS have channeling fractions comparable to those at DAMA, they are still subdominant to other null experiment constraints.
- [39] In reality, even this parameterization is still a qualitative approximation, and one expects that at any recoil energy there is a distribution of q_{chan} 's.